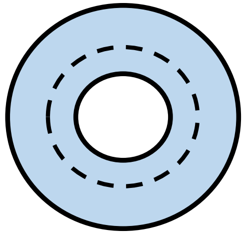
Fluxoid Quantization:

From Prof. D. van Harlingen, Phys 498, UIUC
https://courses.engr.illinois.edu/phys498sqd/fa2019/course-description.html



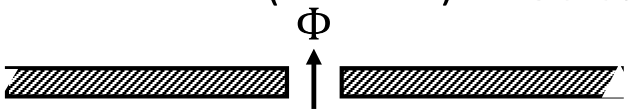
$$\Phi' \;=\; \Phi + \frac{mc}{n_s e^2}\oint \vec{J}_s \cdot \overrightarrow{d\ell} = n\Phi_0 \;=\; \Phi + \frac{mc}{e}\oint \vec{v}_s \cdot \overrightarrow{d\ell}$$

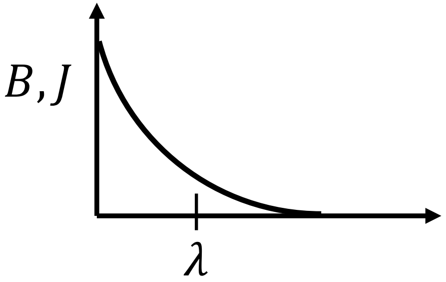
$\vec{J}_s = 0 \;\Rightarrow\; \Phi = n\Phi_0$ flux quantization

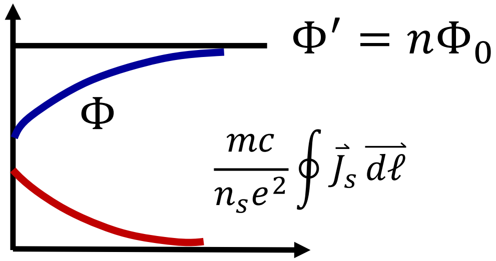
$\vec{J}_s \neq 0 \;\Rightarrow\; \Phi' = n\Phi_0$ flux is not quantized

SITUATIONS where this applies

(1) Near surfaces (within λ) $\quad SC$ sample with trapped flux







(2) Near vortex core (within λ)

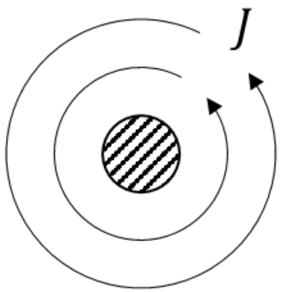
(3) Thin samples ($w < \lambda$) $\Rightarrow$ Little-Parks experiment

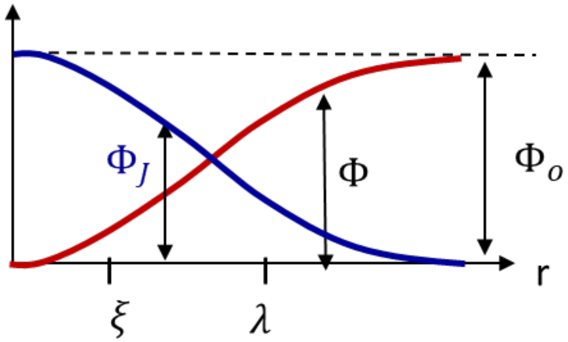
(4) Rotating sample $\Rightarrow$ London rotation

(5) Transport current $\vec{J}_n$ $\Rightarrow$ thermoelectric effect

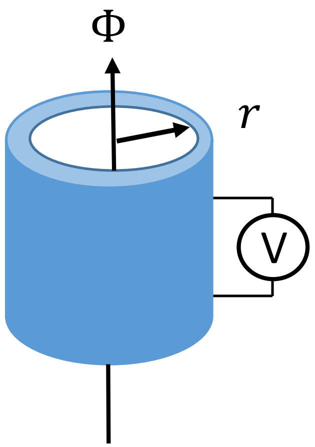
(6) SC weak links $\Rightarrow$ Josephson effect

(7) Unconventional SC $\Rightarrow$ d-wave, p-wave symmetry



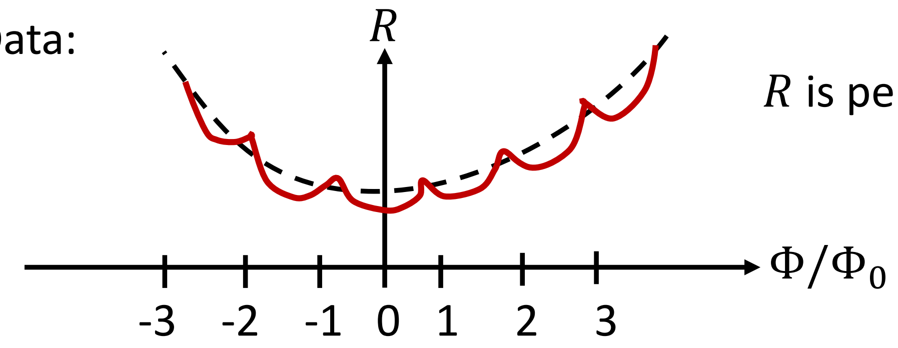


(3) Little-Parks Experiment (1962)



Measure resistance of a superconductor cylinder just above T_c

Data:



R is periodic in Φ

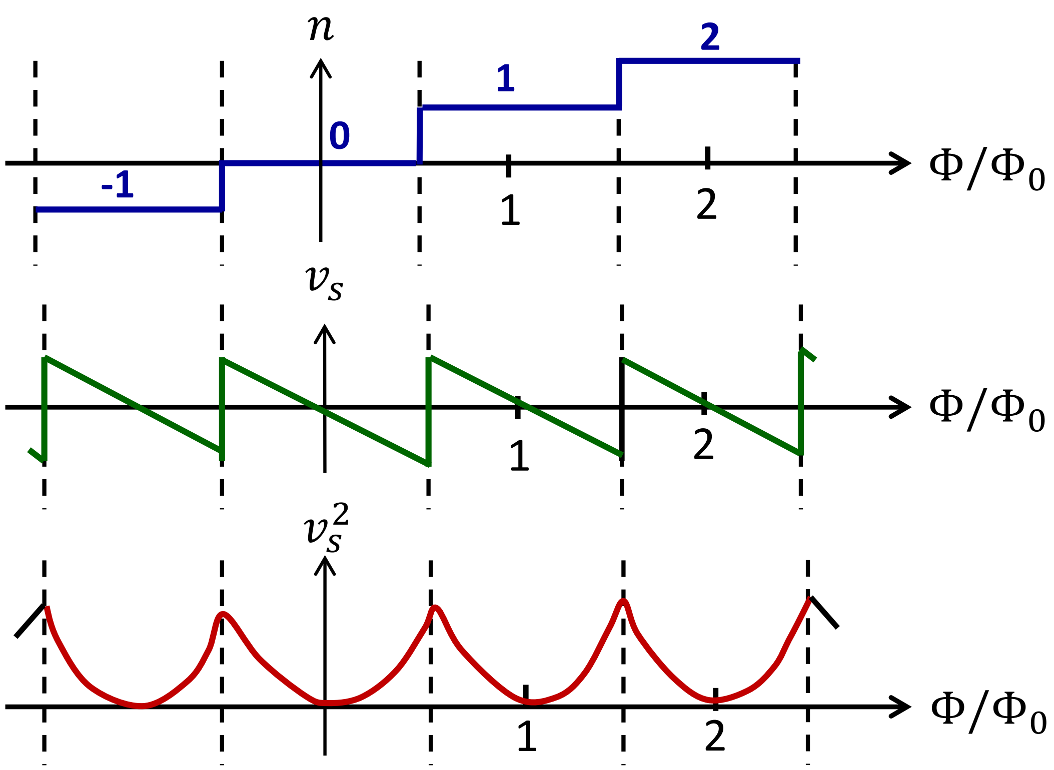
Model:

$$\Phi' = n\Phi_0 = \Phi + \frac{mc}{e}\oint \vec{v}_s\, \overrightarrow{d\ell} = \Phi + \frac{mc}{c}(2\pi r)v_s$$

$$v_s = \frac{\hbar}{mr}\left(n - \frac{\Phi}{\Phi_0}\right)$$

v_s determined by n and Φ --- maximize ΔG $\Rightarrow$

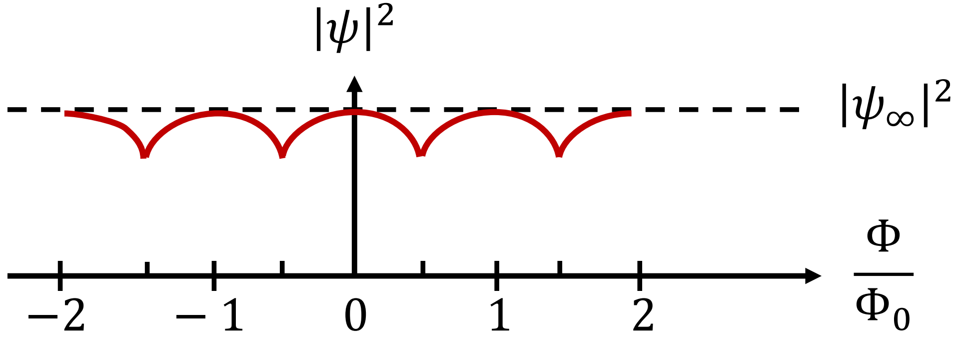
$|v_s|$ small as possible: $\Delta G \sim n_s\left(\frac{1}{2}m^* v_s^2\right)$



Recall variation of $|\psi|$ with v_s:

$$|\psi|^2 = |\psi_\infty|^2 \left[1 - \left(\frac{\xi m^* v_s}{\hbar}\right)^2\right]$$

$$= |\psi_\infty|^2 \left[1 - \left(\frac{2\xi}{r}\right)^2 \left(n - \frac{\Phi}{\Phi_0}\right)^2\right]$$



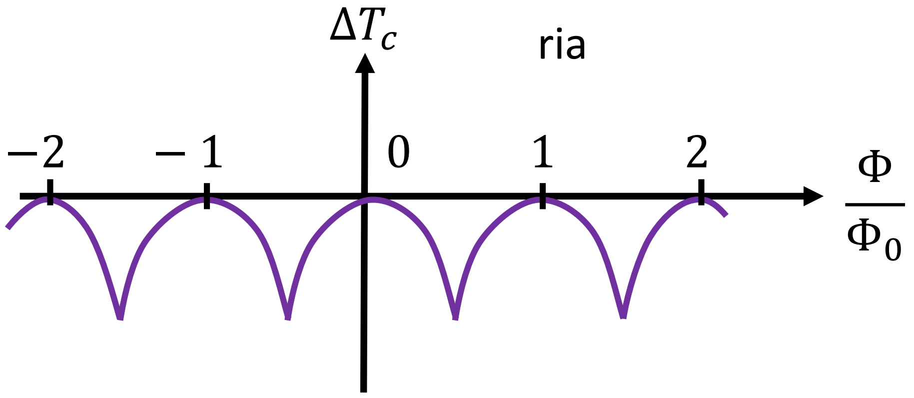
Transition temperature T_c when $|\psi|^2 \to 0 \quad (d \ll \xi, \lambda)$

$$\frac{1}{\xi^2} = \left(\frac{2}{r}\right)^2 \left(n - \frac{\Phi}{\Phi_0}\right)^2 \sim \frac{1-t}{\xi_0^2}$$

$$\xi \sim \frac{\xi_0}{(1-t)^{1/2}}$$

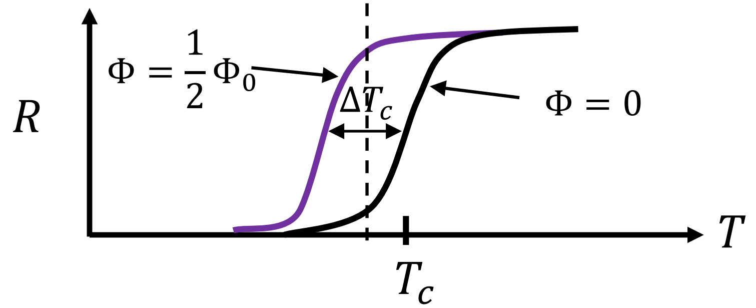
$$1 - t = \frac{\Delta T_c}{T_c} \sim \left(\frac{2\xi_0}{r}\right)^2 \left(n - \frac{\Phi}{\Phi_0}\right)^2$$

$$t = \frac{T}{T_c}$$



Max suppression is $\frac{\Delta T_c}{T_c} \sim \left(\frac{\xi_0}{r}\right)^2$ at $\Phi = \left(n + \frac{1}{2}\right)\Phi_0$

Shows up experimentally as a variation in R (at constant T):

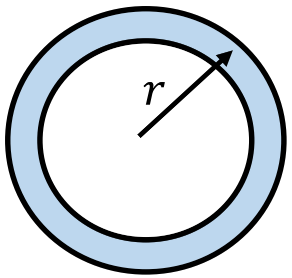


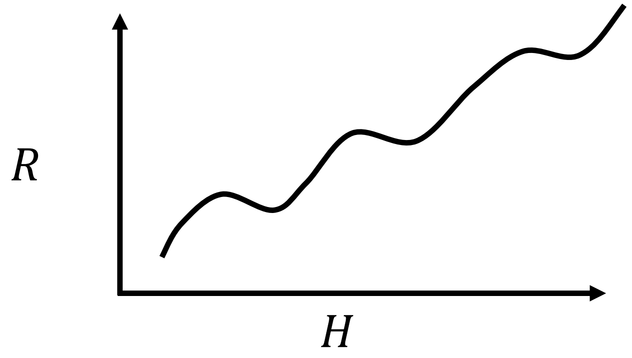
Significance of Little-Parks experiments:

(1) showed reality of the "fluxoid"

(2) demonstrated use of GL free energy to understand experiments

Sharvin & Sharvin repeated this experiment for nanoscale normal rings:





Observed quantized magnetoresistance oscillations for

$$2\pi r < \ell_\phi = v_F \tau_\phi$$

phase coherence length in N $(< 1\mu m)$

This is due to phase coherence in normal metals over microscopic scales, not superconductivity

Important result in nanoscale physics